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tables a more exact calculation is necessary. I have made considerable progress in the calculation of accurate tables, and hope soon to have them completed.

Cape of Good Hope:
1890 *May* 5.

On the Computation of the Equation of the Centre in Elliptical Orbits of Moderate Eccentricities. By A. Marth.

Though the indirect solution of Kepler's problem is a sufficiently easy proceeding when only a limited number of positions have to be computed, it takes rather too much time and becomes tiresome when it has to be repeated periodically hundreds of times. To provide for such cases some method is required for deducing the true anomalies directly from the mean anomalies with the desired degree of accuracy.

To find the true anomalies with the accuracy sufficient for approximate ephemerides of the minor planets, two sets of general tables have been computed: Mr. Godward's "Auxiliary Tables for Computing an Approximate Ephemeris of a Minor Planet" . . . printed 1866, and used in the *Nautical Almanac* office, and MM. Callandreau and Fabry's "Tables numériques destinées a faciliter le calcul des éphemerides" . . . published, 1885, in the second volume of the *Bulletin Astronomique.*

To get the true anomalies with an accuracy required for closer computations, the late E. Schubert gives in his later tables and papers on the elements of the minor planets a special "Table for the correction to be added to the auxiliary anomaly v'," this v' being found by the equation

$$\cot \tfrac{1}{2} v' = \frac{1-e}{1+e} \cot \tfrac{1}{2} \text{ mean anomaly.}$$

The present paper is intended to furnish the means for facilitating the calculation of certain tables, which will allow the differences between the true and mean anomalies to be easily deduced, whenever the intended work may make the construction of such tables desirable.

If e denotes the eccentricity, μ and v the mean and true anomaly, ω the angular value of the radius, the equation of the centre is given by the known series (all terms to e^{10} inclusive being retained) as follows:—

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$$\begin{aligned} v-\mu = {} & 2\omega\left(1-\frac{1}{8}e^2+\frac{5}{192}e^4+\frac{107}{9216}e^6+\frac{6217}{737280}e^8+\ldots\right).e\sin\mu. \\ & +\frac{5}{4}\omega\left(1-\frac{11}{30}e^2+\frac{17}{240}e^4+\frac{43}{7200}e^6+\frac{677}{86400}e^8+\ldots\right).e^2\sin 2\mu. \\ & +\frac{13}{12}\omega\left(1-\frac{129}{208}e^2+\frac{285}{1664}e^4-\frac{973}{66560}e^6+\quad\ldots\right).e^3\sin 3\mu. \\ & +\frac{103}{96}\omega\left(1-\frac{451}{515}e^2+\frac{4123}{12360}e^4-\frac{1619}{25956}e^6+\ldots\right).e^4\sin 4\mu. \\ & +\frac{1097}{960}\omega\left(1-\frac{29785}{26328}e^2+\frac{824605}{1474368}e^4-\ldots\right).e^5\sin 5\mu. \\ & +\frac{1223}{960}\omega\left(1-\frac{23739}{17122}e^2+\frac{116265}{136976}e^4-\quad\ldots\right).e^6\sin 6\mu. \\ & +\frac{47273}{32256}\omega\left(1-\frac{12412897}{7563680}e^2+\ldots\right).e^7\sin 7\mu. \\ & +\frac{556403}{322560}\omega\left(1-\frac{9490966}{5007627}e^2+\ldots\right).e^8\sin 8\mu. \\ & +\frac{10661993}{5160960}\omega\,.\,e^9\sin 9\mu+\frac{7281587}{2903040}\omega\,.\,e^{10}\sin 10\mu+\ldots \\ & =k_1\,.\,e\sin\mu+k_2\,.\,e^2\sin 2\mu+k_3\,.\,e^3\sin 3\mu+k_4\,.\,e^4\sin 4\mu+\ldots \\ & =k_1\,.\,e\sin\mu\,.\,(1+\kappa\,.\,e\cos\mu)+k_3\,.\,e^3\sin 3\mu+k_4\,.\,e^4\sin 4\mu+\ldots \end{aligned}$$

where $\kappa=\frac{5}{4}\left(1-\frac{29}{120}e^2+\frac{7}{480}e^4+\frac{571}{230400}e^6+\frac{23657}{11059200}e^8\right).$

If a table containing the values of

$$k_3\,.\,e^3\sin 3\mu+k_4\,.\,e^4\sin 4\mu+\ldots$$

were prepared, the equation of the centre would be found by computing $k_1.\,e\sin\mu\,(1+\kappa\,.\,e\cos\mu)$ for each individual mean anomaly, and adding the corresponding tabular value representing the remainder of the series. However, as this remainder would be obviously smaller in a series in which the first term, instead of being multiplied by a factor $1+\kappa e\cos\mu$, had a divisor of the form $1-ce\cos\mu$, it will clearly be of advantage to convert the series for the equation of the centre into another, beginning with a term of the form

$$\frac{Ce\sin\mu}{1-ce\cos\mu},$$

in which the values of C and c are so determined that the first two terms of the old series are incorporated in the new expression. These values are accordingly to be found.

$$\frac{\sin\mu}{1-ce\cos\mu}=\sin\mu\,(1+ce\cos\mu+c^2e^2\cos^2\mu+c^3e^3\cos^3\mu+c^4e^4\cos^4\mu+\ldots)$$

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But

$$\begin{aligned}
4 \sin\mu \cos^2\mu &= \sin\mu + \sin 3\mu. \\
16 \sin\mu \cos^4\mu &= 2 \sin\mu + 3 \sin 3\mu + \sin 5\mu. \\
64 \sin\mu \cos^6\mu &= 5 \sin\mu + 9 \sin 3\mu + 5 \sin 5\mu + \sin 7\mu. \\
256 \sin\mu \cos^8\mu &= 14 \sin\mu + 28 \sin 3\mu + 20 \sin 5\mu + 7 \sin 7\mu + \sin 9\mu. \\
8 \sin\mu \cos^3\mu &= 2 \sin 2\mu + \sin 4\mu. \\
32 \sin\mu \cos^5\mu &= 5 \sin 2\mu + 4 \sin 4\mu + \sin 6\mu. \\
128 \sin\mu \cos^7\mu &= 14 \sin 2\mu + 14 \sin 4\mu + 6 \sin 6\mu + \sin 8\mu. \\
512 \sin\mu \cos^9\mu &= 42 \sin 2\mu + 48 \sin 4\mu + 27 \sin 6\mu + 8 \sin 8\mu + \sin 10\mu.
\end{aligned}$$

Hence, putting for greater simplicity $ce = 2\eta$,

$$\begin{aligned}
\frac{\sin\mu}{1 - ce\cos\mu} = &\,(1 + \eta^2 + 2\eta^4 + 5\eta^6 + 14\eta^8 + \ldots) \,.\, \sin\mu. \\
&+ (1 + 2\eta^2 + 5\eta^4 + 14\eta^6 + 42\eta^8 + \ldots) \,.\, \eta \sin 2\mu. \\
&+ (1 + 3\eta^2 + 9\eta^4 + 28\eta^6 + \ldots) \,.\, \eta^2 \sin 3\mu. \\
&+ (1 + 4\eta^2 + 14\eta^4 + 48\eta^6 + \ldots) \,.\, \eta^3 \sin 4\mu. \\
&+ (1 + 5\eta^2 + 20\eta^4 + \ldots) \,.\, \eta^4 \sin 5\mu. \\
&+ (1 + 6\eta^2 + 27\eta^4 + \ldots) \,.\, \eta^5 \sin 6\mu. \\
&+ (1 + 7\eta^2 + \ldots) \,.\, \eta^6 \sin 7\mu. \\
&+ (1 + 8\eta^2 + \ldots) \,.\, \eta^7 \sin 8\mu + \eta^8 \sin 9\mu + \eta^9 \sin 10\mu + \ldots
\end{aligned}$$

and therefore

writing θ for $\eta\,(1 + \eta^2 + 2\eta^4 + 5\eta^6 + 14\eta^8 + \ldots)$,

$$\frac{Ce\sin\mu}{1 - ce\cos\mu} = \frac{2C}{c}\,(\theta\sin\mu + \theta^2\sin 2\mu + \theta^3\sin 3\mu + \theta^4\sin 4\mu + \ldots).$$

The value of c must accordingly be so chosen that

$$\theta = \frac{k_2}{k_1} \,.\, e = \frac{5}{8}e\left(1 - \frac{29}{120}e^2 + \frac{7}{480}e^4 + \frac{571}{230400}e^6 + \frac{23657}{11059200}e^8\right),$$

and C so that

$$\frac{2C}{c} \,.\, \theta = k_1.$$

The determination of the two values which satisfy these conditions gives—

$$C = 2\omega\left(1 - \frac{33}{64}e^2 + \frac{1705}{4096}e^4 - \frac{666617}{2359296}e^6 + \frac{18106717}{83886080}e^8 \ldots\right).$$

$$c = \frac{5}{4}\left(1 - \frac{607}{960}e^2 + \frac{27671}{61440}e^4 - \frac{19289249}{58982400}e^6 + \frac{2688060283}{11324620800}e^8 \ldots\right).$$

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Hence follows—

$$\frac{Ce\sin\mu}{1-ce\cos\mu}=2\omega\left(1-\frac{1}{8}e^2+\frac{5}{192}e^4+\frac{107}{9216}e^6+\frac{6217}{737280}e^8+\ldots\right).\,e\sin\mu.$$

$$+\frac{5}{4}\omega\left(1-\frac{11}{30}e^2+\frac{17}{240}e^4+\frac{43}{7200}e^6+\frac{677}{86400}e^8+\ldots\right).\,e^2\sin 2\mu.$$

$$+\frac{25}{32}\omega\left(1-\frac{73}{120}e^2+\frac{1253}{7200}e^4-\frac{3229}{230400}e^6\ldots\right).\,e^3\sin 3\mu.$$

$$+\frac{125}{256}\omega\left(1-\frac{17}{20}e^2+\frac{537}{1600}e^4-\frac{107939}{1728000}e^6\ldots\right).\,e^4\sin 4\mu.$$

$$+\frac{625}{2048}\omega\left(1-\frac{131}{120}e^2+\frac{889}{1600}e^4\ldots\right).\,e^5\sin 5\mu.$$

$$+\frac{3125}{16384}\omega\left(1-\frac{4}{3}e^2+\frac{1201}{1440}e^4\ldots\right).\,e^6\sin 6\mu.$$

$$+\frac{15625}{131072}\omega\left(1-\frac{63}{40}e^2\ldots\right).\,e^7\sin 7\mu.$$

$$+\frac{78125}{1048576}\omega\left(1-\frac{109}{60}e^2\ \ldots\right).\,e^8\sin 8\mu.$$

$$+\frac{390625}{8388608}\omega\,.\,e^9\sin 9\mu+\frac{1953125}{67108864}\omega\,.\,e^{10}\sin 10\mu+\ \ldots$$

the substitution of which in the old series for the equation of the centre furnishes the new series

$$v-\mu=\frac{Ce\sin\mu}{1-ce\cos\mu}+\{c_3e^3\sin 3\mu+c_4e^4\sin 4\mu+\ \ldots\ c_{10}e^{10}\sin 10\mu\},$$

in which the coefficients $c_3, c_4, c_5 \ldots$ have the following values

$$c_3=\frac{29}{96}\omega\left(1-\frac{151}{232}e^2+\frac{457}{2784}e^4-\frac{7207}{445440}e^6\ldots\right).$$

$$c_4=\frac{449}{768}\omega\left(1-\frac{8057}{8980}e^2+\frac{143047}{431040}e^4-\frac{300761}{4827648}e^6\ldots\right).$$

$$c_5=\frac{25729}{30720}\omega\left(1-\frac{707495}{617496}e^2+\frac{19386485}{34579776}e^4\ldots\right).$$

$$c_6=\frac{266213}{245760}\omega\left(1-\frac{2601092}{1863491}e^2+\frac{152311165}{178895136}e^4\ldots\right).$$

$$c_7=\frac{11117513}{8257536}\omega\left(1-\frac{732409783}{444700520}e^2\ldots\right).$$

$$c_8=\frac{545147297}{330301440}\omega\left(1-\frac{37265543611}{19625302692}e^2\ldots\right).$$

$$c_9=\frac{5335893541}{2642411520}\omega.$$

$$c_{10}=\frac{471668976257}{190253629440}\omega.$$

A comparison of these values with the values of the corresponding coefficients of the old series shows the advantage of the new series in the diminution of the coefficients.

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The logarithms of the coefficients are, if m is the modulus,

$$\log C = \log 2\,\omega - \frac{33}{64}\,m\,.\,e^2 + \frac{2321}{8192}\,m\,.\,e^4 - \frac{268043}{2359296}\,m\,.\,e^6 + \frac{77029999}{1006632960}\,m\,.\,e^8.$$

$$\log c = \log \frac{5}{4} - \frac{607}{960}\,m\,.\,e^2 + \frac{461681}{1843200}\,m\,.\,e^4 - \frac{335831383}{2654208000}\,m\,.\,e^6$$
$$+ \frac{235312199009}{3397386240000}\,m\,.\,e^8$$

$$\log c_3 = \log\left(\frac{29}{96}\,\omega\right) - \frac{151}{232}\,m\,.\,e^2 - \frac{15391}{322944}\,m\,.\,e^4 - \frac{466537}{374615040}\,m\,.\,e^6.$$

$$\log c_4 = \log\left(\frac{449}{768}\,\omega\right) - \frac{8057}{8980}\,m\,.\,e^2 - \frac{68350979}{967684800}\,m\,.\,e^4 - \frac{214816003651}{40552444352000}\,m\,.\,e^6.$$

$$\log c_5 = \log\left(\frac{25729}{30720}\,\omega\right) - \frac{707495}{617496}\,m\,.\,e^2 - [8{\cdot}98109]\,m\,.\,e^4.$$

$$\log c_6 = \log\left(\frac{266213}{245760}\,\omega\right) - \frac{2601092}{1863491}\,m\,.\,e^2 - [9{\cdot}08903]\,m\,.\,e^4.$$

or numerically, if ω is expressed in degrees,

$$\log C = 2{\cdot}059152628 - [9{\cdot}3501183]\,e^2 + [9{\cdot}0900695]\,e^4 - [8{\cdot}69321]\,e^6$$
$$+ [8{\cdot}52157]\,e^8.$$

$$\log c = 0{\cdot}096910013 - [9{\cdot}4387018]\,e^2 + [9{\cdot}0365539]\,e^4 - [8{\cdot}73997]\,e^6$$
$$+ [8{\cdot}47828]\,e^8.$$

$$\log c_3 = 1{\cdot}2382494 - [9{\cdot}4512733]\,e^2 - [8{\cdot}31592]\,e^4 - [6{\cdot}73309]\,e^6.$$

$$\log c_4 = 1{\cdot}5250078 - [9{\cdot}5906814]\,e^2 - [8{\cdot}48680]\,e^4 - [7{\cdot}36183]\,e^6.$$

$$\log c_5 = 1{\cdot}6811243 - [9{\cdot}6968735]\,e^2 - [8{\cdot}61887]\,e^4.$$

$$\log c_6 = 1{\cdot}7928407 - [9{\cdot}7826127]\,e^2 - [8{\cdot}72682]\,e^4.$$

$$\log c_7 = 1{\cdot}88728 - [9{\cdot}8545]\,e^2.$$

$$\log c_8 = 1{\cdot}97573 - [9{\cdot}9163]\,e^2.$$

$$\log c_9 = 2{\cdot}06333.$$

$$\log c_{10} = 2{\cdot}15243.$$

Instead of the coefficients of e^2 . . . their logarithms are given enclosed in [ ] brackets, 10 being added to the characteristics.

If the values of C, c_3, c_4 . . . are required to be expressed in seconds, log 3600 must be added to the first terms, which become thus—

$$\log C = 5{\cdot}615455129 - \ldots$$

$$\log c_3 = 4{\cdot}7945519 \quad - \ldots$$

$$\log c_4 = 5{\cdot}0813103 \quad - \ldots$$

$$\log c_5 = 5.2374268 - \ldots$$

$$\log c_6 = 5{\cdot}3491432 - \ldots$$

$$\log c_7 = 5{\cdot}44358.$$

I have given all terms to e^{10} inclusive, so that, if needed, they may be taken into account. The following table contains, for various values of log e, the values of the coefficients $c_7 e^7$, $c_8 e^8$, $c_9 e^9$, $c_{10} e^{10}$ expressed in degrees and also in seconds.

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Coefficients expressed

	In Degrees.				In Seconds of Arc.			
log e	c_7e^7	c_8e^8	c_9e^9	$c_{10}e^{10}$	c_7e^7	c_8e^8	c_9e^9	$c_{10}e^{10}$
	°				″	″	″	″
9·00	0·000008	·000001	...	...	0·03	0·003	...	...
·02	010	001	...	...	·04	·005	...	...
·04	014	002	...	...	·05	·007	...	...
·06	020	003	...	...	·07	·010	...	...
·08	027	004	...	...	·10	·014	...	...
9·10	·000038	·000006	001	...	0·14	0·021	0·003	...
·12	052	008	001	...	·19	·030	·005	...
·14	071	012	002	...	·26	·043	·008	...
·16	098	017	003	...	·35	·062	·011	...
·18	135	025	005	...	·49	·090	·017	...
9·20	·000186	·000036	007	001	0·67	0·13	0·026	0·005
·22	256	052	011	002	0·92	·19	·040	·008
·24	351	074	017	004	1·26	·27	·060	·013
·26	483	107	025	006	1·74	·38	·091	·020
·28	663	153	038	009	2·39	·55	·14	·03
9·30	·000909	·000220	058	014	3·27	0·79	0·21	0·05
·32	·001248	316	088	023	4·49	1·14	0·32	0·08
·34	·001710	453	133	036	6·16	1·63	0·48	0·13
·36	·002343	649	201	057	8·43	2·34	0·72	0·20
·38	·003207	930	304	090	11·55	3·35	1·10	0·32
9·40	0·004387	·001330	461	142	15·79	4·79	1·66	0·51

In case the desired accuracy allows the neglect of $c_7e^7 \sin 7\mu$, the computation of the tabular values of

$$\{c_3 e^3 \sin 3\mu + c_4 e^4 \sin 4\mu + c_5 e^5 \sin 5\mu + c_6 e^6 \sin 6\mu\}$$

for any value of log e and for each degree of μ is easily accomplished, if the required log sines of the multiples of μ are specially written out beforehand, so that the 30 + 22 + 18 + 15 numerical values, which enter into combination, may be quickly computed.

If that is done for conveniently altered values of log e, the variation of the eccentricity may be easily taken into account. To save all trouble in computing the coefficients, their logarithms for suitable values of log e are supplied by the following tables.

In the computation of

$$\frac{Ce \sin \mu}{1 - ce \cos \mu}$$

the values of log C and log c must be properly interpolated to correspond to the varied values of log e. Want of space did not permit the differences of successive values to be given on the last pages.

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log e	log C		log c		log c_3e^3	log c_4e^4	log c_5e^5
8·00	2·059 1302	21	0·096 8826	27	5·238		
·02	1381	24	8799	29	5·298		
·04	1257	26	8770	32	5·358		
·06	1231	28	8738	35	5·418		
·08	1203	32	8703	38	5·478		
8·10	2·059 1171	34	0·096 8665	42	5·538		
·12	1137	37	8623	46	5·598		
·14	1100	42	8577	51	5·658		
·16	1058	45	8426	55	5·718		
·18	1013	49	8471	61	5·778		
8·20	2·059 0964	54	0·096 8410	66	5·8382		
·22	0910	60	8344	73	5·8982		
·24	0850	65	8271	80	5·9582		
·26	0785	72	8191	88	6·0182		
·28	0713	78	8103	96	6·0781		
8·30	2·059 0635	86	0·096 8007	105	6·1381	4·725	
·32	0549	94	7902	116	6·1981	4·805	
·34	0455	104	7786	127	6·2581	4·885	
·36	0351	113	7659	139	6·3181	4·965	
·38	0238	124	7520	152	6·3781	5·045	
8·40	2·059 0114	136	0·096 7368	167	6·4381	5·125	
·42	·058 9978	150	7101	183	6·4981	5·205	
·44	9828	164	7018	201	6·5580	5·285	
·46	9664	179	6817	220	6·6180	5·365	
·48	9485	197	6597	242	6·6780	5·445	
8·50	2·058 9288	216	0·096 6355	265	6·7380	5·525	
·52	9072	236	6090	290	6·7979	5·605	
·54	8836	260	5800	318	6·8579	5·685	
·56	8576	284	5482	349	6·9179	5·764	
·58	8292	312	5133	382	6·9778	5·844	
8·60	2·058 7980	342	0·096 4751	420	7·0378	5·9244	4·680
·62	7638	374	4331	459	7·0978	6·0043	4·780
·64	7264	411	3872	504	7·1577	6·0843	4·880
·66	6853	450	3368	553	7·2177	6·1642	4·980
·68	6403	494	2815	606	7·2776	6·2441	5·080
8·70	2·058 5909	541	0·096 2209	664	7·3375	6·3240	5·180
·72	5368	593	1545	728	7·3975	6·4039	5·280
·74	4775	650	0817	798	7·4574	6·4838	5·380
·76	4125	713	·096 0019	875	7·5173	6·5637	5·479
·78	3412	781	·095 9144	959	7·5772	6·6436	5·579
8·80	2·058 2631		0·095 8185		7·6371	6·7235	5·679

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log e	log C	log c	log c_3e^3	log c_4e^4	log c_5e^5	log c_6e^6
8·80	2·058 2631	0·095 8185	7·6371	6·7235	5·679	4·590
·81	·058 2213	·095 7672	7·6671	6·7634	5·729	4·650
·82	·058 1775	·095 7134	7·6970	6·8033	5 779	4·710
·83	·058 1316	·095 6571	7·7270	6·8432	5·829	4·770
·84	·058 0836	·095 5982	7·7569	6·8831	5·879	4·830
8·85	2·058 0334	0·095 5365	7·7868	6·9231	5·929	4·890
·86	·057 9808	·095 4719	7·8168	6·9630	5·9785	4·950
·87	·057 9257	·095 4042	7·8467	7·0029	6·0284	5·010
·88	·057 8681	·095 3334	7·8766	7·0428	6·0783	5·069
·89	·057 8078	·095 2593	7·9065	7·0827	6·1281	5·129
8·90	2·057 7446	0·095 1817	7·9365	7·1225	6·1780	5·189
·91	·057 6785	·095 1005	7·9664	7·1624	6·2278	5·249
·92	·057 6093	·095 0154	7·9963	7·2023	6·2777	5·309
·93	·057 5368	·094 9264	8·02620	7·2422	6·3275	5·368
·94	·057 4610	·094 8332	8·05610	7·2820	6·3773	5·428
8·95	2·057 3816	0·094 7356	8·08600	7·3219	6·4272	5·488
·96	·057 2985	·094 6335	8 11590	7·3618	6·4770	5·548
·97	·057 2116	·094 5266	8·14579	7·4016	6·5270	5·608
·98	·057 1205	·094 4146	8·17567	7·4415	6·5766	5·667
8·99	·057 0253	·094 2975	8·20555	7·4813	6·6264	5·727
9·00	2·056 9256	0·094 1748	8·23542	7·5211	6·6761	5·787
·01	·056 8212	·094 0465	8·26529	7·5609	6·7259	5·846
·02	·056 7120	·093 9121	8·29515	7·6007	6·7757	5·906
·03	·056 5977	·093 7714	8·32500	7·6405	6·8254	5·966
·04	·056 4781	·093 6242	8·35485	7·6803	6 8751	6·0255
9·05	2·056 3529	0·093 4701	8·38469	7·7201	6·9249	6·0852
·06	·056 2219	·093 3089	8·41452	7·7599	6·9746	6 1448
·07	·056 0848	·093 1401	8·44434	7·7996	7·0242	6·2045
·08	·055 9414	·092 9634	8·47416	7·8394	7·0739	6·2641
·09	·055 7913	·092 7785	8·50396	7·8791	7·1236	6·3237
9·10	2·055 6342	0·092 5850	8·53376	7·9188	7·1732	6·3832
·11	·055 4699	·092 3825	8·56355	7·9585	7·2229	6·4428
·12	·055 2980	·092 1706	8·59333	7·99823	7·2725	6·5023
·13	·055 1182	·091 9488	8·62310	8·03791	7·3221	6·5618
·14	·054 9300	·091 7167	8·65286	8 07757	7·3716	6 6213
9·15	2·054 7332	0 091 4739	8·68260	8 11722	7·4212	6·6807
·16	·054 5273	·091 2198	8·71233	8·15685	7·4707	6·7402
·17	·054 3119	·090 9539	8·74205	8·19647	7·5202	6·7996
·18	·054 0866	·090 6757	8·77176	8·23607	7·5697	6·8589
·19	·053 8510	·090 3846	8·80146	8·27564	7·6192	6·9183
9·20	2·053 6046	0·090 0801	8·83114	8·31520	7 6686	6·9776

log e	log C	log c	log c_3e^3	log c_4e^4	log c_5e^5	log c_6e^6
9·200	2·053 6046	0·090 0801	8·83114	8·31520	7·6686	6·9776
·205	·053 4771	·089 9226	8·84597	8·33497	7·6933	7·0072
·210	·053 3468	·089 7615	8·86080	8·35474	7·7180	7·0369
·215	·053 2136	·089 5968	8·87563	8·37450	7·7427	7·0665
·220	·053 0773	·089 4283	8·89045	8·39425	7·7674	7·0961
·225	·052 9380	·089 2559	8·90527	8·41400	7·7921	7·1257
·230	·052 7955	·089 0796	8·92008	8·43374	7·8167	7·1553
·235	·052 6498	·088 8993	8·93489	8·45348	7·8414	7·1849
·240	·052 5008	·088 7149	8·94969	8·47321	7·8661	7·2145
·245	·052 34[illegible]5	·088 5263	8·96449	8·49294	7·8907	7·2441
9·250	2·052 1928	0·088 3334	8·97929	8·51266	7·9153	7·2736
·255	·052 0335	·088 1362	8·994081	8·53237	7·9400	7·3032
·260	·051 8707	·087 9344	9·008867	8·55207	7·9646	7·3327
·265	·051 7042	·087 7281	9·023648	8·57177	7·9892	7·3622
·270	·051 5340	·087 5171	9·038423	8·59146	8·01382	7·3918
·275	·051 3599	·087 3013	9·053194	8·61114	8·03842	7·4213
·280	·051 1820	·087 0807	9·067959	8·63082	8·06300	7·4508
·285	·051 0001	·086 8550	9·082719	8·65049	8·08758	7·4802
·290	·050 8141	·086 6243	9·097473	8·67015	8·11215	7·5097
·295	·050 6240	·086 3883	9·112221	8·68980	8·13670	7·5392
9·300	2·050 4297	0·086 1470	9·126963	8·70945	8·16125	7·5686
·305	·050 2310	·085 9002	9·141700	8·72908	8·18578	7·5981
·310	·050 0279	·085 6479	9·156430	8·74871	8·21031	7·6275
·315	·049 8203	·085 3899	9·171154	8·76833	8·23482	7·6569
·320	·049 6081	·085 1261	9·185871	8·78794	8·25932	7·6863
·325	·049 3911	·084 8563	9·200582	8·80754	8·28381	7·7157
·330	·049 1694	·084 5805	9·215286	8·82713	8·30829	7·7450
·335	·048 9428	·084 2985	9·229983	8·84671	8·33276	7·7744
·340	·048 7112	·084 0101	9·244673	8·86629	8·35721	7·8037
·345	·048 4744	·083 7153	9·259355	8·88585	8·38165	7·8330
9·350	2·048 2325	0·083 4139	9·274030	8·90540	8·40608	7·8623
·355	·047 9852	·083 1057	9·288698	8·92494	8·43050	7·8916
·360	·047 7325	·082 7906	9·303358	8·94447	8·45490	7·9209
·365	·047 4742	·082 4685	9·318010	8·96399	8·47928	7·9501
·370	·047 2103	·082 1392	9·332653	8·98360	8·50365	7·9794
·375	·046 9406	·081 8026	9·347288	9·002998	8·52801	8·00858
·380	·046 6650	·081 4584	9·361915	9·022483	8·55235	8·03778
·385	·046 3834	·081 1066	9·376633	9·041956	8·57668	8·06696
·390	·046 0957	·080 7470	9·391142	9·060417	8·60099	8·09612
·395	·045 8017	·080 3794	9·405742	9·080865	8·62528	8·12526
9·400	2·045 5014	0·080 0036	9·420332	9·100299	8·64956	8·15438

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A simple Solution of Kepler's Problem. By A. Marth.

What is Kepler's problem? Is it the problem to find the eccentric from the mean anomaly *by means* of Kepler's equation? Or is it the problem to find somehow the eccentric anomaly which will *satisfy* that equation?

The simple method, which I now offer, for computing the eccentric anomaly ϵ from the mean anomaly μ, consists in substituting for Kepler's equation $\epsilon-\mu=e \sin \epsilon$ an equivalent equation, the solution of which is free from the difficulties with which that of Kepler's equation is beset, and which is derived from the latter by an extremely simple process.

The multiplication of

$$\epsilon-\mu=e \sin \epsilon \text{ by } \frac{\sin(\epsilon-\mu)}{\epsilon-\mu}$$

gives

$$\sin \epsilon \cos \mu - \cos \epsilon \sin \mu = e \sin \epsilon \, . \, \frac{\sin(\epsilon-\mu)}{\epsilon-\mu};$$

Hence

$$\tan \epsilon = \frac{\sin \mu}{\cos \mu - e \, . \, \dfrac{\sin(\epsilon-\mu)}{\epsilon-\mu}}$$

or

$$\tan \epsilon = \frac{\sin \mu}{\cos \mu - \dfrac{e}{\nu}}$$

if

$$\frac{1}{\nu} = \frac{\sin(e \sin \epsilon)}{e \sin \epsilon}$$

$$= 1 - \frac{e^2 \sin^2 \epsilon}{6} + \frac{e^4 \sin^4 \epsilon}{120} - \frac{e^6 \sin^6 \epsilon}{5040} + \frac{e^8 \sin^8 \epsilon}{362880} - \frac{e^{10} \sin^{10} \epsilon}{39916800} + \ldots$$

The equation $\mu = \epsilon - e \sin \epsilon$ retains its value for computing μ from ϵ, and may thus serve as a control upon the determination of $\tan \epsilon$. Except for the purpose of such control, the angle ϵ itself is not required.

The system of formulæ for deducing the true anomaly v and the radius vector ρ from the mean anomaly μ becomes

$$\left.\begin{array}{l} \tan \epsilon = \dfrac{\sin \mu}{\cos \mu - \dfrac{e}{\nu}} \\ \rho \sin v = \sin \epsilon \, . \, \sqrt{1-e^2} \\ \rho \cos v = \cos \epsilon - e. \end{array}\right\}$$

The values of $\log \nu$ corresponding to $\log (e \sin \epsilon)$ will be given in a table in the next number of the *Monthly Notices.*